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CENTER FOR THE APPLICATION OF MATHEMATICS
Lehigh University
Bethlehem, Pa.

PHOTOELASTICITY WITH FINITE DEFORMATIONS

by

G. F. Smith

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R. S. Rivlin

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Abstract

A phenomenological theory is developed for the propagation of plane electromagnetic waves in a deformed non-absorbing centrosymmetric isotropic material. It is assumed that the dielectric constant and specific reluctance matrices depend on the deformation gradients at the instant of measurement. The theory is formulated from both the Eulerian and Lagrangian standpoints.

1. Introduction

In this paper we consider the propagation of plane electromagnetic waves in a non-absorbing material which is subjected to finite deformations. It is assumed that the material is isotropic when undeformed and when no electromagnetic fields are present and that it is centrosymmetric. The theory is formulated from both Eulerian and Lagrangian points-of-view. The latter formulation rests on the Lagrangian formulation of Maxwell's equations for a deformed material due to Walker, Pipkin and Rivlin [1].

In each case the assumption is made that the material is linear with respect to electromagnetic effects, but that the dielectric constant and specific reluctance matrices may depend on the displacement gradients in the material. It follows from the isotropic character of the material that the dielectric constant and specific reluctance matrices are isotropic matrix functions of the Cauchy and Finger strains, accordingly as the Lagrangian or Eulerian formulation is adopted and may be expressed in terms of these in canonical forms. In each case we obtain from the constitutive equations and Maxwell's equations a secular equation for the determination of the slowness of a plane electromagnetic wave, propagating in an arbitrary direction in a material which is subjected to a pure homogeneous deformation.

We pursue the study of this equation in the Eulerian case and obtain the six principal slownesses. It is found that there is a relation between these six slownesses. In the case when only the dielectric constant or only the specific reluctance depends on the deformation, this single relation is replaced by three relations.

We then discuss the propagation of the electromagnetic wave in any direction in a principal plane. In §§ 5 and 6 we consider propagation in a material which is subjected to shearing deformations.

Finally in § 7 we consider the application of the theory to materials for which the dielectric constant and specific reluctance matrices depend on the history of the deformation, but in which the deformation is held constant.

2. The constitutive equations

(a) Eulerian formulation

We consider a body to undergo a deformation which is described in a rectangular cartesian coordinate system x by

$$x_i = x_i(t) = x_i(X_A, t), \tag{2.1}$$

where x_i is the position in the system x, at time t, of a particle which was at X_A in the same system at a reference time t_o.

We make the constitutive assumption that the electric displacement field $\bar{d}_i$, at time t, depends only on the electric field $\bar{e}_p$ and deformation gradients $x_{p,A}$, measured at the particle considered at time t. We also assume that the dependence of $\bar{d}_i$ on $\bar{e}_p$ is linear. We make the analogous constitutive assumption that the magnetic induction field $\bar{b}_i$, at time t, depends only on the magnetic intensity field $\bar{h}_p$ and deformation gradients $x_{p,A}$, the dependence on the former being linear.

If the material is isotropic in its reference state, it follows [2] that

$$\bar{d}_i = k_{ij}\bar{e}_j \text{ and } \bar{h}_i = \omega_{ij}\bar{b}_j, \tag{2.2}$$

where k_{ij}, the dielectric constant tensor, and ω_{ij}, the specific reluctance tensor, are given by

$$k_{ij} = k_o\delta_{ij} + k_1 c_{ij} + k_2 c_{ik}c_{kj}$$

and (2.3)

$$\omega_{ij} = \omega_o\delta_{ij} + \omega_1 c_{ij} + \omega_2 c_{ik}c_{kj},$$

where c_{ij} is the Finger strain tensor defined by

$$c_{ij} = x_{i,A}x_{j,A} - \delta_{ij}. \tag{2.4}$$

In (2.3), k_o, k_1, k_2, ω_o, ω_1, ω_2 are functions of the invariants tr $\underset{\sim}{c}$, tr $\underset{\sim}{c}^2$, tr $\underset{\sim}{c}^3$, where $\underset{\sim}{c} = \|c_{ij}\|$. Introducing the notation $\underset{\sim}{k} = \|k_{ij}\|$, $\underset{\sim}{e} = (\bar{e}_i)$, with analogous meanings for other bold-face symbols, we may rewrite (2.2) as

$$\bar{\underset{\sim}{d}} = \underset{\sim}{k}.\bar{\underset{\sim}{e}} \text{ and } \bar{\underset{\sim}{h}} = \underset{\sim}{\omega}.\bar{\underset{\sim}{b}}, \tag{2.5}$$

where

$$\begin{aligned} \underset{\sim}{k} &= k_o\underset{\sim}{I} + k_1\underset{\sim}{c} + k_2\underset{\sim}{c}^2, \\ \underset{\sim}{\omega} &= \omega_o\underset{\sim}{I} + \omega_1\underset{\sim}{c} + \omega_2\underset{\sim}{c}^2, \end{aligned} \tag{2.6}$$

and $\underset{\sim}{I}$ denotes the unit matrix.

For a plane electromagnetic wave, adopting the usual complex notation, we may write $\bar{\underset{\sim}{e}}$, $\bar{\underset{\sim}{h}}$, $\bar{\underset{\sim}{d}}$, $\bar{\underset{\sim}{b}}$ in the form

$$(\bar{\underset{\sim}{e}}, \bar{\underset{\sim}{h}}, \bar{\underset{\sim}{d}}, \bar{\underset{\sim}{b}}) = (\underset{\sim}{e}, \underset{\sim}{h}, \underset{\sim}{d}, \underset{\sim}{b})\, e^{i\omega(\underset{\sim}{s}.\underset{\sim}{x}-t)}, \tag{2.7}$$

where $\underset{\sim}{e}$, $\underset{\sim}{h}$, $\underset{\sim}{d}$, $\underset{\sim}{b}$ and $\underset{\sim}{s}$ are vectors which may be real, imaginary, or complex constants.† $\underset{\sim}{s}$ is the complex slowness of the wave and ω is its angular frequency. Then, the constitutive equations (2.5) become

$$\underset{\sim}{d} = \underset{\sim}{k}.\underset{\sim}{e}, \quad \underset{\sim}{h} = \underset{\sim}{\omega}.\underset{\sim}{b}. \tag{2.8}$$

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†We will see later that for the constitutive equation discussed here, the case when $\underset{\sim}{s}$ is complex can be ruled out.

(b) Lagrangian formulation

An alternative formulation may be attained in the following way. In accord with Walker, Pipkin and Rivlin [1], we define the reduced fields $\bar{\mathbf{E}}$, $\bar{\mathbf{H}}$, $\bar{\mathbf{B}}$, $\bar{\mathbf{D}}$ by the equations

$$\begin{gathered}\bar{\mathbf{E}} = \overset{*}{\mathbf{F}}.\bar{\mathbf{e}},\ \bar{\mathbf{H}} = \overset{*}{\mathbf{F}}.\bar{\mathbf{h}},\\ \bar{\mathbf{B}} = (\det \mathbf{F})\ \mathbf{F}^{-1}.\bar{\mathbf{b}} \text{ and } \bar{\mathbf{D}} = (\det \mathbf{F})\ \mathbf{F}^{-1}.\bar{\mathbf{d}},\end{gathered} \tag{2.9}$$

where the notation

$$\mathbf{F} = \left\| F_{iA} \right\| = \left\| x_{i,A} \right\| \tag{2.10}$$

is used and the star denotes the transpose. The constitutive assumptions made as a basis for the Eulerian formulation are equivalent to the assumptions that $\bar{\mathbf{D}}$ and $\bar{\mathbf{B}}$ are linear functions of $\bar{\mathbf{E}}$ and $\bar{\mathbf{H}}$ respectively and both $\bar{\mathbf{D}}$ and $\bar{\mathbf{B}}$ depend on $\mathbf{F}$.

Then, the assumption that the material is isotropic in its reference state leads to the conclusion that

$$\bar{\mathbf{D}} = \mathbf{K}.\bar{\mathbf{E}} \text{ and } \bar{\mathbf{H}} = \boldsymbol{\Omega}.\bar{\mathbf{B}} \tag{2.11}$$

and $\mathbf{K}$ and $\boldsymbol{\Omega}$ are expressible in the forms

$$\mathbf{K} = K_o\mathbf{I} + K_1\mathbf{C} + K_2\mathbf{C}^2$$

and (2.12)

$$\boldsymbol{\Omega} = \Omega_o\mathbf{I} + \Omega_1\mathbf{C} + \Omega_2\mathbf{C}^2,$$

where $\mathbf{C}$ is the Cauchy strain defined by

$$\underset{\sim}{C} = \left\| C_{AB} \right\| = \left\| x_{i,A} x_{i,B} - \delta_{AB} \right\| \qquad (2.13)$$

and K_o, K_1, K_2, Ω_o, Ω_1, Ω_2 are functions of tr $\underset{\sim}{C}$, tr $\underset{\sim}{C}^2$, tr $\underset{\sim}{C}^3$.

The relations between K_α, Ω_α (α=0,1,2) and k_α, ω_α (α=0,1,2) can be derived. However, the algebra involved is somewhat cumbersome.

Now, we consider the electromagnetic wave for which, adopting the usual complex notation,

$$(\underset{\sim}{\bar{E}}, \underset{\sim}{\bar{H}}, \underset{\sim}{\bar{D}}, \underset{\sim}{\bar{B}}) = (\underset{\sim}{E}, \underset{\sim}{H}, \underset{\sim}{D}, \underset{\sim}{B}) e^{i\omega(\underset{\sim}{S}.\underset{\sim}{X}-t)}, \qquad (2.14)$$

where $\underset{\sim}{E}$, $\underset{\sim}{H}$, $\underset{\sim}{D}$, $\underset{\sim}{B}$ and $\underset{\sim}{S}$ may be real, imaginary or complex constant vectors. We obtain from (2.11)

$$\underset{\sim}{D} = \underset{\sim}{K}.\underset{\sim}{E}, \quad \underset{\sim}{H} = \underset{\sim}{\Omega}.\underset{\sim}{B}. \qquad (2.15)$$

We note that if the electromagnetic fields $\underset{\sim}{\bar{e}}$, $\underset{\sim}{\bar{h}}$, $\underset{\sim}{\bar{d}}$, $\underset{\sim}{\bar{b}}$ correspond to a plane wave, i.e., are of the form (2.7), the derived electromagnetic fields $\underset{\sim}{\bar{E}}$, $\underset{\sim}{\bar{H}}$, $\underset{\sim}{\bar{D}}$, $\underset{\sim}{\bar{B}}$ will not, in general, have the form (2.14).

3. Derivation of the secular equation

(a) Eulerian formulation

Maxwell's equations may be written in the form

$$\text{curl}\ \bar{\underset{\sim}{e}} = -\partial\bar{\underset{\sim}{b}}/\partial t,\ \text{curl}\ \bar{\underset{\sim}{h}} = \partial\bar{\underset{\sim}{d}}/\partial t, \tag{3.1}$$

where

$$(\text{curl}\ \bar{\underset{\sim}{e}})_i = \varepsilon_{ijk}\bar{e}_{k,j}. \tag{3.2}$$

Introducing (2.7), we obtain

$$\varepsilon_{ijk}s_j e_k = b_i,\ \varepsilon_{ijk}s_j h_k = -d_i. \tag{3.3}$$

Eliminating $\underset{\sim}{e}$, $\underset{\sim}{h}$ and $\underset{\sim}{b}$ from equations (3.3) and (2.8), we obtain,

$$\left[\delta_{ij} + \varepsilon_{ipq}\varepsilon_{mrs}s_p s_r \omega_{qm}(\underset{\sim}{k}^{-1})_{sj}\right] d_j = 0. \tag{3.4}$$

Alternatively eliminating $\underset{\sim}{e}$, $\underset{\sim}{h}$ and $\underset{\sim}{d}$ from equations (3.3) and (2.8) we obtain

$$\left[\delta_{ij} + \varepsilon_{ipq}\varepsilon_{mrs}s_p s_r (\underset{\sim}{k}^{-1})_{qm}\omega_{sj}\right] b_j = 0. \tag{3.5}$$

Equation (3.4) yields a non-trivial solution for $\underset{\sim}{d}$ and (3.5) yields a non-trivial solution for $\underset{\sim}{b}$ provided that

$$|k_{ij} + \varepsilon_{ipq}\varepsilon_{jsr}s_p s_r \omega_{qs}| = 0. \tag{3.6}$$

For the wave (2.7), the planes of constant phase are

$$\underset{\sim}{s}^{+}.\underset{\sim}{x} = \text{constant} \tag{3.7}$$

and the planes of constant amplitude are

$$\underset{\sim}{s}^{-}.\underset{\sim}{x} = \text{constant}. \tag{3.8}$$

In the particular case when these are the same, we may write

$$\underset{\sim}{s} = s\underset{\sim}{n}, \tag{3.9}$$

where $\underset{\sim}{n}$ is a (real) unit vector and s is a constant which may be real, imaginary, or complex. Then, equations (3.4) and (3.5) become

$$\left[\delta_{ij} + s^2\varepsilon_{ipq}\varepsilon_{mrs}n_p n_r \omega_{qm}(\underset{\sim}{k}^{-1})_{sj}\right] d_j = 0$$

and (3.10)

$$\left[\delta_{ij} + s^2\varepsilon_{ipq}\varepsilon_{mrs}n_p n_r (\underset{\sim}{k}^{-1})_{qm}\omega_{sj}\right] b_j = 0,$$

and (3.6) yields the secular equation for the complex slowness s,

$$|k_{ij} + s^2\varepsilon_{ipq}\varepsilon_{jsr}n_p n_r \omega_{qs}| = 0. \tag{3.11}$$

We shall call the direction of $\underset{\sim}{n}$ the direction of propagation of the wave.

It is shown in the Appendix that (3.11) may be written as

$$\phi s^4 - \psi s^2 + \theta = 0, \tag{3.12}$$

where

$$\phi = (\underset{\sim}{n}.\underset{\sim}{k}.\underset{\sim}{n})(\underset{\sim}{n}.\underset{\sim}{\omega}^{-1}.\underset{\sim}{n}) \det \underset{\sim}{\omega},$$

$$\psi = \underset{\sim}{n}.\{(\mathrm{tr}\ \underset{\sim}{k}\ \underset{\sim}{\omega})\underset{\sim}{k} - \underset{\sim}{k}\ \underset{\sim}{\omega}\ \underset{\sim}{k}\}.\underset{\sim}{n}, \tag{3.13}$$

$$\theta = \det \underset{\sim}{k}.$$

Equations (3.10) may be simplified slightly by choosing the reference system so that the unit normal to the wave-front is in the direction of the x_3-axis, i.e., so that $n_i = \delta_{i3}$. Equations (3.3) then yield, with (3.9),

$$d_3 = b_3 = 0 \tag{3.14}$$

and equations (3.10) become

$$\left[\delta_{\alpha\beta} - s^2\varepsilon_{\alpha\gamma}\varepsilon_{\rho\tau}\omega_{\gamma\tau}(\underset{\sim}{k}^{-1})_{\rho\beta}\right] d_\beta = 0$$

and (3.15)

$$\left[\delta_{\alpha\beta} - s^2\varepsilon_{\alpha\gamma}\varepsilon_{\rho\tau}(\underset{\sim}{k}^{-1})_{\gamma\tau}\omega_{\rho\beta}\right] b_\beta = 0,$$

where Greek indices take the values 1, 2 and $\varepsilon_{\alpha\beta}$ denotes the two-dimensional alternating symbol.

It is evident from (3.14) and (3.15) that the wave is, in general, polarized elliptically with its electric displacement and magnetic induction fields in planes normal to the direction of propagation. It then follows from (2.8) that $\underset{\sim}{e}$ and $\underset{\sim}{h}$ are, in general, not perpendicular to the direction of the propagation.

Introducing $n_i = \delta_{i3}$ into (3.11), or more simply from (3.15) we can rewrite the secular equation as

$$|k_{\alpha\beta} - s^2\varepsilon_{\alpha\gamma}\varepsilon_{\beta\tau}\omega_{\gamma\tau}| = 0. \tag{3.16}$$

From (3.12), s^2 is given by

$$s^2 = \{\psi\pm(\psi^2 - 4\theta\phi)^{1/2}\}/2\phi. \tag{3.17}$$

These values of s^2 are real if and only if

$$\psi^2 > 4\theta\phi. \tag{3.18}$$

If both of the values of s^2 given by (3.17) are positive, then we obtain two positive values of s and two negative values. This corresponds to the possibility of two waves in the positive direction of $\underset{\sim}{n}$ and two waves in the negative direction. If, on the other hand, ϕ,ψ and θ are such that for any $\underset{\sim}{n}$, one of the values of s^2, given by (3.17), is negative, the corresponding values of s are imaginary. The material would then be inherently electromagnetically unstable in the state of deformation considered.

(b) Lagrangian formulation

It has been pointed out by Walker, Pipkin and Rivlin [1] that in terms of the derived electromagnetic fields $\bar{\underset{\sim}{E}}$, $\bar{\underset{\sim}{H}}$, $\bar{\underset{\sim}{D}}$ and $\bar{\underset{\sim}{B}}$, Maxwell's equations can be written as

$$\text{Curl } \bar{\underset{\sim}{E}} = -\, \partial\bar{\underset{\sim}{B}}/\partial t, \quad \text{Curl } \bar{\underset{\sim}{H}} = \partial\bar{\underset{\sim}{D}}/\partial t, \tag{3.19}$$

where

$$(\text{Curl } \bar{\underset{\sim}{E}})_A = \varepsilon_{ABC}\bar{E}_{C,B}. \tag{3.20}$$

Introducing (2.14) into (3.19), we obtain

$$\varepsilon_{ABC}S_B E_C = B_A, \quad \varepsilon_{ABC}S_B H_C = -D_A. \tag{3.21}$$

Eliminating $\underset{\sim}{E}$, $\underset{\sim}{H}$ and $\underset{\sim}{B}$ from (3.21) and (2.15), we obtain

$$\left[\delta_{AB} + \varepsilon_{APQ}\varepsilon_{MRS}S_P S_R \Omega_{QM}(\underset{\sim}{K}^{-1})_{SB}\right] D_B = 0. \tag{3.22}$$

Again, eliminating $\underset{\sim}{E}$, $\underset{\sim}{H}$ and $\underset{\sim}{D}$ from (3.21) and (2.15) we obtain

$$\left[\delta_{AB} + \varepsilon_{APQ}\varepsilon_{MRS}S_P S_R (\underset{\sim}{K}^{-1})_{QM}\Omega_{SB}\right] B_B = 0. \tag{3.23}$$

Again, if the planes of constant amplitude and phase in the X-space are the same and $\underset{\sim}{N}$ is the unit normal perpendicular to this plane, we may write analogously with (3.9),

$$\underset{\sim}{S} = S\underset{\sim}{N}, \tag{3.24}$$

where S is a constant which may be real, imaginary, or complex. Then, equations (3.22) and (3.23) become

$$\left[\delta_{AB} + S^2 \varepsilon_{APQ}\varepsilon_{MRS} N_P N_R \Omega_{QM} (\underset{\sim}{K}^{-1})_{SB}\right] D_B = 0$$

and (3.25)

$$\left[\delta_{AB} + S^2 \varepsilon_{APQ}\varepsilon_{MRS} N_P N_R (\underset{\sim}{K}^{-1})_{QM} \Omega_{SB}\right] B_B = 0.$$

The secular equation for S is

$$|K_{AB} + S^2 \varepsilon_{APQ}\varepsilon_{BML} N_P N_L \Omega_{QM}| = 0. \qquad (3.26)$$

Following a procedure similar to that used in the Appendix to derive (3.12), we can express (3.22) in the form

$$\Phi S^4 - \Psi S^2 + \Theta = 0, \qquad (3.27)$$

where

$$\Phi = (\underset{\sim}{N}.\underset{\sim}{K}.\underset{\sim}{N})(\underset{\sim}{N}.\underset{\sim}{\Omega}^{-1}.\underset{\sim}{N}) \det \underset{\sim}{\Omega},$$

$$\Psi = \underset{\sim}{N}.\{(\mathrm{tr}\ \underset{\sim}{K}\ \underset{\sim}{\Omega})\ \underset{\sim}{K} - \underset{\sim}{K}\ \underset{\sim}{\Omega}\ \underset{\sim}{K}\}.\underset{\sim}{N}, \qquad (3.28)$$

$$\Theta = \det \underset{\sim}{K}.$$

4. Pure homogeneous deformation

(a) Propagation in principal direction

We now suppose that the deformation to which the body is subjected is the pure homogeneous deformation, the principal directions for which are along the axes of the reference system x. Then

$$c_{ij} = 0 \quad (i \neq j) \tag{4.1}$$

and it follows from (2.3) that

$$k_{ij} = 0, \quad \omega_{ij} = 0 \quad (i \neq j). \tag{4.2}$$

The principal waves are waves for which the directions of propagation are along the principal directions of strain, i.e., the waves for which

$$n_i = \delta_{i1}, \ \delta_{i2}, \text{ or } \delta_{i3}. \tag{4.3}$$

We consider first the waves propagated along the x_3-axis. Then, introducing (4.2) into (3.16), we obtain

$$\begin{vmatrix} k_{11} - s^2\omega_{22}, & 0 \\ 0 & , k_{22} - s^2\omega_{11} \end{vmatrix} = 0, \tag{4.4}$$

whence

$$s^2 = k_{11}/\omega_{22} \text{ or } s^2 = k_{22}/\omega_{11}. \tag{4.5}$$

We assume that these quantities are positive and consider the waves corresponding to the positive square roots, i.e., the waves travelling in the positive direction of the x_3-axis. We employ the notation

$$s_{13} = (k_{11}/\omega_{22})^{1/2}, \; s_{23} = (k_{22}/\omega_{11})^{1/2}. \tag{4.6}$$

We note from (3.15) that, for the wave for which $s = s_{13}$, $d_2 = b_1 = 0$ and, for the wave for which $s = s_{23}$, $d_1 = b_2 = 0$. Thus, the former wave is polarized with $\underset{\sim}{d}$ and $\underset{\sim}{b}$ in the x_1 and x_2 directions respectively and the latter with $\underset{\sim}{d}$ and $\underset{\sim}{b}$ in the x_2 and x_1 directions respectively. It follows from (4.2) that for these waves $\underset{\sim}{e}$ is polarized in the same direction as $\underset{\sim}{d}$ and $\underset{\sim}{h}$ in the same direction as $\underset{\sim}{b}$.

More generally, we adopt the notation that s_{ij} $(i \neq j)$ is the slowness for the principal wave whose direction of propagation is along the x_j-axis and which is polarized with its electric displacement field in the x_i-direction. Then, analogously with (4.6), we have the further relations

$$\begin{aligned} s_{32} &= (k_{33}/\omega_{11})^{1/2}, \; s_{12} = (k_{11}/\omega_{33})^{1/2}, \\ s_{21} &= (k_{22}/\omega_{33})^{1/2}, \; s_{31} = (k_{33}/\omega_{22})^{1/2}. \end{aligned} \tag{4.7}$$

It follows from (4.6) and (4.7) that

$$s_{13}s_{32}s_{21} = s_{23}s_{12}s_{31} \tag{4.8}$$

and this relation is valid for any constitutive equations of the form (2.8) with $\underset{\sim}{k}$ and $\underset{\sim}{\omega}$ given by (2.6), k_0, k_1, k_2 and ω_0, ω_1, ω_2 being arbitrary functions of tr $\underset{\sim}{c}$, tr $\underset{\sim}{c}^2$, tr $\underset{\sim}{c}^3$.

In the case when $\omega_1 = \omega_2 = 0$ and ω_o is constant, i.e. the specific reluctance is independent of deformation, we have $\omega_{11} = \omega_{22} = \omega_{33} = \omega_o$ and it follows from (4.6) and (4.7) that

$$s_{13} = s_{12},\ s_{21} = s_{23},\ s_{32} = s_{31}. \qquad (4.9)$$

If on the other hand $k_1 = k_2 = 0$ and k_o is constant, i.e. the dielectric constant is independent of deformation, we have $k_{11} = k_{22} = k_{33} = k_o$ and it follows from (4.6) and (4.7) that

$$s_{23} = s_{32},\ s_{13} = s_{31},\ s_{21} = s_{12}. \qquad (4.10)$$

(b) Direction of propagation in principal plane

We shall now consider the somewhat more general case when the direction of propagation $\underset{\sim}{n}$ is in the plane formed by two of the principal directions of strain. Choosing the coordinate system x with the x_2-axis perpendicular to this plane, we have

$$n_i = (n_1, 0, n_3), \tag{4.11}$$

and the Finger strain components are, as before, c_{ij}, with $c_{ij} = 0$ $(i \neq j)$. We choose a new coordinate system $\tilde{x}$ with the axis $\tilde{x}_3$ parallel to $\underset{\sim}{n}$ and $\tilde{x}_2$ coinciding with x_2. Then

$$\tilde{x}_i = a_{ij} x_j, \tag{4.12}$$

where a_{ij} is given by

$$\left\| a_{ij} \right\| = \left\| \begin{array}{ccc} n_3, & 0, & -n_1 \\ 0, & 1, & 0 \\ n_1, & 0, & n_3 \end{array} \right\| . \tag{4.13}$$

The components $\tilde{c}_{ij}$ of the Finger strain tensor in the system $\tilde{x}$ are given by

$$\tilde{c}_{ij} = a_{ip} a_{jq} c_{pq}. \tag{4.14}$$

Thus,

$$\left\| \tilde{c}_{ij} \right\| = \left\| \begin{matrix} c_{11}n_3^2 + c_{33}n_1^2, & 0 & , (c_{11}-c_{33})n_1n_3 \\ 0 & , \quad c_{22} & , \quad 0 \\ (c_{11}-c_{33})n_1n_3, & 0 & , c_{11}n_1^2 + c_{33}n_3^2 \end{matrix} \right\|$$

and (4.15)

$$\left\| \tilde{c}_{ik}\tilde{c}_{kj} \right\| = \left\| \begin{matrix} c_{11}^2n_3^2 + c_{33}^2n_1^2, & 0 & , (c_{11}^2-c_{33}^2)n_1n_3 \\ 0 & , \quad c_{22}^2 & , \quad 0 \\ (c_{11}^2-c_{33}^2)n_1n_3, & 0 & , c_{11}^2n_1^2 + c_{33}^2n_3^2 \end{matrix} \right\| .$$

Referred to the system $\tilde{x}$, the dielectric constant and inverse magnetic permeability matrices, $\underset{\sim}{\tilde{k}}$ and $\underset{\sim}{\tilde{\omega}}$ respectively, are, by analogy with the relations (2.3), given by

$$\tilde{k}_{ij} = k_o\delta_{ij} + k_1\tilde{c}_{ij} + k_2\tilde{c}_{ik}\tilde{c}_{kj}$$

and (4.16)

$$\tilde{\omega}_{ij} = \omega_o\delta_{ij} + \omega_1\tilde{c}_{ij} + \omega_2\tilde{c}_{ik}\tilde{c}_{kj},$$

where k_o, k_1, ..., ω_2 have precisely the same meanings as in (2.3). Since tr $\underset{\sim}{c}$, tr $\underset{\sim}{c}^2$, tr $\underset{\sim}{c}^3$ are invariant under orthogonal transformations, k_o, ..., ω_2 may be expressed as functions of tr $\tilde{\underset{\sim}{c}}$, tr $\tilde{\underset{\sim}{c}}^2$, tr $\tilde{\underset{\sim}{c}}^3$ of the same forms as they are functions of tr $\underset{\sim}{c}$, tr $\underset{\sim}{c}^2$, tr $\underset{\sim}{c}^3$. Introducing (4.15) into (4.16), we see that

$$\tilde{k}_{ij} = (\tilde{\underset{\sim}{k}}^{-1})_{ij} = \tilde{\omega}_{ij} = (\tilde{\underset{\sim}{\omega}}^{-1})_{ij} = 0,$$

$$(ij = 12,\ 21,\ 23,\ 32). \quad (4.17)$$

Introducing (4.17) into the secular equation (3.16), with $\underset{\sim}{k}$ and $\underset{\sim}{\omega}$ replaced by $\tilde{\underset{\sim}{k}}$ and $\tilde{\underset{\sim}{\omega}}$ respectively, we obtain

$$s^2 = \tilde{s}_{13}^2 \text{ (say)} = \tilde{k}_{11}/\tilde{\omega}_{22}$$

or (4.18)

$$s^2 = \tilde{s}_{23}^2 \text{ (say)} = \tilde{k}_{22}/\tilde{\omega}_{11}.$$

Again assuming $\tilde{k}_{11}/\tilde{\omega}_{22}$ and $\tilde{k}_{22}/\tilde{\omega}_{11}$ are both positive, we see from (3.15), with $\underset{\sim}{k}$, $\underset{\sim}{\omega}$, $\underset{\sim}{d}$ and $\underset{\sim}{b}$ replaced by $\tilde{\underset{\sim}{k}}$, $\tilde{\underset{\sim}{\omega}}$, $\tilde{\underset{\sim}{d}}$ and $\tilde{\underset{\sim}{b}}$ respectively, that for the wave for which $s = \tilde{s}_{13}$, $\tilde{d}_2 = \tilde{b}_1 = 0$ and for the wave for which $s = \tilde{s}_{23}$, $\tilde{d}_1 = \tilde{b}_2 = 0$.

5. Simple shear

We now consider the case when the deformation is a simple shear of amount κ, described in the rectangular cartesian coordinate system x by

$$x_1 = X_1,\ x_2 = X_2,\ x_3 = X_3 + \kappa X_1. \tag{5.1}$$

We consider a wave propagated in the positive direction of the x_3-axis, so that

$$n_i = \delta_{i3}. \tag{5.2}$$

From (5.1) and (2.4), we obtain

$$\left\| c_{ij} \right\| = \left\| \begin{matrix} 0, & 0, & \kappa \\ 0, & 0, & 0 \\ \kappa, & 0, & \kappa^2 \end{matrix} \right\| , \quad \left\| c_{ik}c_{kj} \right\| = \left\| \begin{matrix} \kappa^2, & 0, & \kappa^3 \\ 0, & 0, & 0 \\ \kappa^3, & 0, & \kappa^2+\kappa^4 \end{matrix} \right\| . \tag{5.3}$$

Introducing (5.3) into (2.3) we obtain

$$\left\| k_{ij} \right\| = \left\| \begin{matrix} k_0+k_2\kappa^2 & , & 0 & , & \kappa(k_1+k_2\kappa^2) \\ 0 & , & k_0 & , & 0 \\ \kappa(k_1+k_2\kappa^2), & & 0 & , & k_0+\kappa^2(k_1+k_2+k_2\kappa^2) \end{matrix} \right\|$$

and (5.4)

$$\left\| \omega_{ij} \right\| = \left\| \begin{matrix} \omega_0+\omega_2\kappa^2 & , & 0 & , & \kappa(\omega_1+\omega_2\kappa^2) \\ 0 & , & \omega_0 & , & 0 \\ \kappa(\omega_1+\omega_2\kappa^2), & & 0 & , & \omega_0+\kappa^2(\omega_1+\omega_2+\omega_2\kappa^2) \end{matrix} \right\| .$$

We recall that k_o, k_1, k_2, ω_o, ω_1, ω_2 are functions of $\mathrm{tr}\,\underset{\sim}{c}$, $\mathrm{tr}\,\underset{\sim}{c}^2$, $\mathrm{tr}\,\underset{\sim}{c}^3$ which are now given by

$$\mathrm{tr}\,\underset{\sim}{c} = \kappa^2, \quad \mathrm{tr}\,\underset{\sim}{c}^2 = 2\kappa^2 + \kappa^4, \quad \mathrm{tr}\,\underset{\sim}{c}^3 = 3\kappa^4 + \kappa^6. \tag{5.5}$$

Introducing (5.4) into (3.16), we obtain

$$s = \left[(k_o+k_2\kappa^2)/\omega_o\right]^{1/2} \text{ or } \left[k_o/(\omega_o+\omega_2\kappa^2)\right]^{1/2}. \tag{5.6}$$

From (3.15) we see that the first of these values of s leads to

$$d_2 = b_1 = 0 \tag{5.7}$$

and the second leads to

$$d_1 = b_2 = 0. \tag{5.8}$$

Thus, in the case provided by $(5.6)_1$, $\underset{\sim}{d}$ and $\underset{\sim}{b}$ are linearly polarized in the x_1 and x_2 directions respectively and in the case provided by $(5.6)_2$, they are linearly polarized in the x_2 and x_1 directions respectively. The corresponding expressions for $\underset{\sim}{e}$ and $\underset{\sim}{h}$ can be obtained by introducing (5.4) and (3.14) into (2.8). We obtain, corrresponding to (5.7),

$$\underset{\sim}{e} = \text{const.}\left[k_o+\kappa^2(k_1+k_2+k_2\kappa^2),\ 0, -\kappa(k_1+k_2\kappa^2)\right],$$

and (5.9)

$$\underset{\sim}{h} = \text{const.}\left[0,\ 1,\ 0\right]$$

and, corresponding to (5.8), we obtain

$$\underset{\sim}{e} = \text{const.} \begin{bmatrix}0, 1, 0\end{bmatrix}$$

and (5.10)

$$\underset{\sim}{h} = \text{const.} \begin{bmatrix}\omega_o+\omega_2\kappa^2, 0, \kappa(\omega_1+\omega_2\kappa^2)\end{bmatrix} .$$

6. Shear in two directions

We now consider the case in which the direction of propagation of the electromagnetic wave is along the x_3-axis, but the direction of shear is in the x_2x_3 plane, the deformation being described by

$$x_1 = X_1, \; x_2 = X_2 + \lambda X_1, \; x_3 = X_3 + \kappa X_1. \tag{6.1}$$

Introducing (6.1) into (2.4), we obtain

$$\left\| c_{ij} \right\| = \left\| \begin{matrix} 0, & \lambda, & \kappa \\ \lambda, & \lambda^2, & \lambda\kappa \\ \kappa, & \lambda\kappa, & \kappa^2 \end{matrix} \right\|$$

and (6.2)

$$\left\| c_{ik}c_{kj} \right\| = \left\| \begin{matrix} \lambda^2+\kappa^2, & 0, & 0 \\ 0, & \lambda^2, & \lambda\kappa \\ 0, & \lambda\kappa, & \kappa^2 \end{matrix} \right\| + (\lambda^2+\kappa^2) \left\| \begin{matrix} 0, & \lambda, & \kappa \\ \lambda, & \lambda^2, & \lambda\kappa \\ \kappa, & \lambda\kappa, & \kappa^2 \end{matrix} \right\| .$$

It follows from (6.2) and (2.3) that

$$\left\| k_{\alpha\beta} \right\| = \left\| \begin{matrix} k_o + k_2(\lambda^2+\kappa^2), & k_1\lambda + k_2\lambda(\lambda^2+\kappa^2) \\ k_1\lambda + k_2\lambda(\lambda^2+\kappa^2), & k_o + k_1\lambda^2 + k_2\lambda^2(1+\lambda^2+\kappa^2) \end{matrix} \right\|$$

and (6.3)

$$\left\|\omega_{\alpha\beta}\right\| = \left\| \begin{matrix} \omega_o + \omega_2(\lambda^2+\kappa^2), & \omega_1\lambda + \omega_2\lambda(\lambda^2+\kappa^2) \\ \omega_1\lambda + \omega_2\lambda(\lambda^2+\kappa^2), & \omega_o + \omega_1\lambda^2 + \omega_2\lambda^2(1+\lambda^2+\kappa^2) \end{matrix} \right\| .$$

k_o, k_1, ..., ω_2 are functions of tr $\underset{\sim}{c}$, tr $\underset{\sim}{c}^2$ and tr $\underset{\sim}{c}^3$, which are given by

$$\begin{aligned} \operatorname{tr} \underset{\sim}{c} &= \lambda^2+\kappa^2 , \quad \operatorname{tr} \underset{\sim}{c}^2 = 2(\lambda^2+\kappa^2) + (\lambda^2+\kappa^2)^2 , \\ \operatorname{tr} \underset{\sim}{c}^3 &= 3(\lambda^2+\kappa^2)^2 + (\lambda^2+\kappa^2)^3. \end{aligned} \tag{6.4}$$

Since the wave considered is propagated along the x_3 axis, it follows that the relations (3.14) and (3.15) are satisfied. We have, therefore,

$$d_3 = b_3 = 0,$$

and (6.5)

$$(\delta_{\alpha\beta}-s^2A_{\alpha\beta})d_\beta = 0 ,$$

where

$$\begin{aligned} A_{11} &= \omega_{22}(\underset{\sim}{k}^{-1})_{11} - \omega_{21}(\underset{\sim}{k}^{-1})_{21}, \quad A_{12}= \omega_{22}(\underset{\sim}{k}^{-1})_{12} - \omega_{21}(\underset{\sim}{k}^{-1})_{22} \\ A_{21} &= \omega_{11}(\underset{\sim}{k}^{-1})_{21} - \omega_{12}(\underset{\sim}{k}^{-1})_{11}, \quad A_{22}= \omega_{11}(\underset{\sim}{k}^{-1})_{22} - \omega_{12}(\underset{\sim}{k}^{-1})_{12}. \end{aligned} \tag{6.6}$$

From $(6.5)_2$, we see that s is given by

$$\left|\delta_{\alpha\beta} - s^2A_{\alpha\beta}\right| = 0, \tag{6.7}$$

i.e.

$$s^{-2} = \tfrac{1}{2}(A_{11}+A_{22}) \pm \tfrac{1}{2} \left\{(A_{11}-A_{22})^2 + 4A_{12}A_{21}\right\}^{1/2}. \tag{6.8}$$

Introducing (6.8) into $(6.5)_2$, we obtain

$$\frac{d_2}{d_1} = \frac{(A_{22}-A_{11}) \pm C^{1/2}}{2A_{12}}, \tag{6.9}$$

where

$$C = (A_{11}-A_{22})^2 + 4A_{12}A_{21}. \tag{6.10}$$

In determining $\underset{\sim}{k}^{-1}$, we may with advantage employ the relation

$$\underset{\sim}{k}^{-1} = \frac{1}{\det \underset{\sim}{k}}\left\{\underset{\sim}{k}^2-(\mathrm{tr}\ \underset{\sim}{k})\ \underset{\sim}{k} + \frac{1}{2}\left[(\mathrm{tr}\ \underset{\sim}{k})^2 - \mathrm{tr}\ \underset{\sim}{k}^2\right]\underset{\sim}{I}\right\} \tag{6.11}$$

which follows directly from the Cayley-Hamilton theorem. Introducing $(2.6)_1$ into (6.11), again employing the Cayley-Hamilton theorem and using the relation

$$\det \underset{\sim}{k} = \frac{1}{6}\left[(\mathrm{tr}\ \underset{\sim}{k})^3 - 3\mathrm{tr}\ \underset{\sim}{k}\ \mathrm{tr}\ \underset{\sim}{k}^2 + 2\mathrm{tr}\ \underset{\sim}{k}^3\right], \tag{6.12}$$

we see that the relation (6.11) may be expressed in the form

$$\underset{\sim}{k}^{-1} = \tau_0 + \tau_1\ \underset{\sim}{c} + \tau_2\underset{\sim}{c}^2, \tag{6.13}$$

where τ_0, τ_1 and τ_2 are expressible as functions of tr $\underset{\sim}{c}$, tr $\underset{\sim}{c}^2$ and tr $\underset{\sim}{c}^3$. Introducing (6.2) into (6.13), we obtain

$$
\begin{aligned}
(\underset{\sim}{k}^{-1})_{11} &= \tau_o + \tau_2(\lambda^2+\kappa^2), \\
(\underset{\sim}{k}^{-1})_{22} &= \tau_o + \tau_1\lambda^2 + \tau_2\lambda^2(1+\lambda^2+\kappa^2), \qquad (6.14) \\
(\underset{\sim}{k}^{-1})_{12} &= (\underset{\sim}{k}^{-1})_{21} = \tau_1\lambda + \tau_2\lambda(\lambda^2+\kappa^2).
\end{aligned}
$$

Introducing (6.14) and (6.3) into (6.6), we obtain

$$
\begin{aligned}
A_{11} &= (\omega_o\tau_o+\omega_o\tau_2\kappa^2) + \lambda^2(\omega_1\tau_o+\omega_2\tau_o-\omega_1\tau_1+\omega_o\tau_2) \\
&\quad + \lambda^2\kappa^2(\omega_2\tau_o+\omega_2\tau_2-\omega_2\tau_1) + \lambda^4\omega_2(\tau_o+\tau_2-\tau_1), \\
A_{22} &= (\omega_o\tau_o+\omega_2\tau_o\kappa^2) + \lambda^2(\omega_o\tau_1+\omega_o\tau_2-\omega_1\tau_1+\omega_2\tau_o) \\
&\quad + \lambda^2\kappa^2(\omega_o\tau_2+\omega_2\tau_2-\omega_1\tau_2) + \lambda^4(\omega_o\tau_2+\omega_2\tau_2-\omega_1\tau_2), \\
A_{12} &= \lambda(\omega_o\tau_1-\omega_1\tau_o) + \lambda\kappa^2(\omega_o\tau_2-\omega_2\tau_o) \qquad (6.15) \\
&\quad + \lambda^3(\omega_2\tau_1-\omega_1\tau_2+\omega_o\tau_2-\omega_2\tau_o), \\
A_{21} &= \lambda(\omega_o\tau_1-\omega_1\tau_o) + \lambda\kappa^2(\omega_o\tau_2-\omega_2\tau_o+\omega_2\tau_1-\omega_1\tau_2) \\
&\quad + \lambda^3(\omega_o\tau_2-\omega_2\tau_o+\omega_2\tau_1-\omega_1\tau_2).
\end{aligned}
$$

For brevity we introduce the notation

$$
\begin{aligned}
A_{11} &= a_{11} + b_{11}\lambda^2 + c_{11}\lambda^4, \\
A_{12} &= a_{12}\lambda + b_{12}\lambda^3, \\
A_{21} &= a_{21}\lambda + b_{21}\lambda^3, \qquad (6.16) \\
A_{22} &= a_{22} + b_{22}\lambda^2 + c_{22}\lambda^4,
\end{aligned}
$$

where, comparing (6.16) and (6.15),

$$a_{11} = \omega_0\tau_0 + \omega_0\tau_2\kappa^2,$$

$$b_{11} = (\omega_1\tau_0+\omega_2\tau_0-\omega_1\tau_1+\omega_0\tau_2) + \kappa^2(\omega_2\tau_0+\omega_2\tau_2-\omega_2\tau_1),$$

$$\ldots \text{ etc.} \tag{6.17}$$

Introducing (6.16) into (6.8) , denoting by s_1 and s_2 the values of s obtained by taking the positive and negative square roots respectively in (6.8), and expanding the expressions for s_1 and s_2 as power series in λ, we obtain

$$\begin{aligned} s_1^{-2} &= a_{11} + b_{11}\lambda^2 + a_{12}a_{21}(a_{11}-a_{22})^{-1}\lambda^2 + \ldots , \\ s_2^{-2} &= a_{22} + b_{22}\lambda^2 - a_{12}a_{21}(a_{11}-a_{22})^{-1}\lambda^2 + \ldots . \end{aligned} \tag{6.18}$$

Introducing (6.16) into (6.9) and expanding the expression obtained as a power series in λ, we obtain

$$\begin{aligned} (d_2/d_1)_{s=s_1} &= \lambda a_{21}(a_{11}-a_{22})^{-1} + \ldots \\ (d_1/d_2)_{s=s_2} &= \lambda a_{12}(a_{22}-a_{11})^{-1} + \ldots . \end{aligned} \tag{6.19}$$

7. Application to time-dependent materials

We now consider that the dielectric constant matrix $\underset{\sim}{k}$ and specific reluctance matrix $\underset{\sim}{\omega}$ depend not only on the deformation gradients existing at the instant of measurement, but on the whole history of the deformation gradients in the particle considered up to this time. This means that $\underset{\sim}{k}$ and $\underset{\sim}{\omega}$ are matrix functionals of the history of the deformation gradients. However, if we restrict the deformations considered to ones in which the body is taken from the undeformed state to a certain state of deformation at some instant of time and then held there, and we further make appropriate assumptions regarding the nature of the functional dependence of $\underset{\sim}{k}$ and $\underset{\sim}{\omega}$ on the deformation gradient history and the path by which the material is taken from its undeformed state to the steady state of deformations, we can still write the constitutive equations for $\underset{\sim}{\overline{d}}$ and $\underset{\sim}{\overline{h}}$ in the forms (2.5) where $\underset{\sim}{k}$ and $\underset{\sim}{\omega}$ are now functions of the steady state deformation gradients and the time which has elapsed since these were produced in the material. All the results obtained in the paper then follow with the proviso that ω_0, ω_1, ω_2 and k_0, k_1, k_2 depend on this time as well as on the strain invariants tr $\underset{\sim}{c}$, tr $\underset{\sim}{c}^2$ and tr $\underset{\sim}{c}^3$.

This parallels the application of results in finite elasticity theory to problems involving stress relaxation in viscoelastic solids held at constant deformation [3,4].

8. Appendix

In this section, we outline the computation which yields the secular equation (3.12). With (3.11), we have

$$\det\,(k_{ij}+s^2\varepsilon_{ipq}\varepsilon_{jrs}n_pn_s\omega_{qr}) = 0. \tag{8.1}$$

Let

$$\alpha_{ij} = s^2\varepsilon_{ipq}\varepsilon_{jrs}n_pn_s\omega_{qr}. \tag{8.2}$$

With (8.2), we may write (8.1) as

$$6\det\,(k_{ij}+\alpha_{ij}) = \varepsilon_{ipq}\varepsilon_{jrs}(k_{ij}+\alpha_{ij})(k_{pr}+\alpha_{pr})\,(k_{qs}+\alpha_{qs}) \tag{8.3}$$

$$= \varepsilon_{ipq}\varepsilon_{jrs}\,(k_{ij}k_{pr}k_{qs}+3k_{ij}k_{pr}\alpha_{qs}+3k_{ij}\alpha_{pr}\alpha_{qs}+\alpha_{ij}\alpha_{pr}\alpha_{qs}) = 0.$$

We then have

$$\varepsilon_{ipq}\varepsilon_{jrs}k_{ij}k_{pr}k_{qs} = 6\det\,\underset{\sim}{k}. \tag{8.4}$$

With (8.2), we obtain

$$3\varepsilon_{ipq}\varepsilon_{jrs}k_{ij}k_{pr}\alpha_{qs} = 6s^2\left[\underset{\sim}{n}.(\underset{\sim}{k}\underset{\sim}{\omega}\underset{\sim}{k}).\underset{\sim}{n} - (\underset{\sim}{n}.\underset{\sim}{k}.\underset{\sim}{n}.)(\mathrm{tr}\,\underset{\sim}{k}\underset{\sim}{\omega})\right], \tag{8.5}$$

$$3\varepsilon_{ipq}\varepsilon_{jrs}k_{ij}\alpha_{pr}\alpha_{qs} = 3s^4(\underset{\sim}{n}.\underset{\sim}{k}.\underset{\sim}{n})\,\Big[2\underset{\sim}{n}.\underset{\sim}{\omega}^2.\underset{\sim}{n}+(\mathrm{tr}\,\underset{\sim}{\omega})^2 - \mathrm{tr}\,\underset{\sim}{\omega}^2 - 2(\mathrm{tr}\,\underset{\sim}{\omega})(\underset{\sim}{n}.\underset{\sim}{\omega}.\underset{\sim}{n})\Big], \tag{8.6}$$

and

$$\varepsilon_{ipq}\varepsilon_{jrs}\alpha_{ij}\alpha_{pr}\alpha_{qs} = 6\det\,\alpha_{ij} = 0. \tag{8.7}$$

With (6.11), we note that the expression appearing within the brackets in (8.6) may be written as

$$\underset{\sim}{n}.\left[2\underset{\sim}{\omega}^2-(\mathrm{tr}\ \underset{\sim}{\omega})\underset{\sim}{\omega}+(\mathrm{tr}\ \underset{\sim}{\omega})^2\underset{\sim}{I}-(\mathrm{tr}\ \underset{\sim}{\omega}^2)\underset{\sim}{I}\right].\underset{\sim}{n} = 2\det \underset{\sim}{\omega}(\underset{\sim}{n}.\underset{\sim}{\omega}^{-1}.\underset{\sim}{n}). \tag{8.8}$$

Substituting (8.4), ..., (8.8) into (8.3) we obtain

$$\det (k_{ij}+\alpha_{ij}) = \phi S^4 - \psi S^2 + \theta = 0 \tag{8.9}$$

where

$$\begin{aligned} \phi &= (\underset{\sim}{n}.\underset{\sim}{k}.\underset{\sim}{n})(\underset{\sim}{n}.\underset{\sim}{\omega}^{-1}.\underset{\sim}{n}) \det \underset{\sim}{\omega}, \\ \psi &= (\underset{\sim}{n}.\underset{\sim}{k}.\underset{\sim}{n})\,\mathrm{tr}\ \underset{\sim}{k}\underset{\sim}{\omega} - \underset{\sim}{n}.\underset{\sim}{k}\underset{\sim}{\omega}\underset{\sim}{k}.\underset{\sim}{n}, \\ \theta &= \det \underset{\sim}{k}. \end{aligned} \tag{8.10}$$

Equations (8.9) and (8.10) yield the result (3.12) and (3.13).

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13. ABSTRACT

A phenomenological theory is developed for the propagation of plane electromagnetic waves in a deformed non-absorbing centrosymmetric isotropic material. It is assumed that the dielectric constant and specific reluctance matrices depend on the deformation gradients at the instant of measurement. The theory is formulated from both the Eulerian and Lagrangian standpoints. ( )

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